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Derivation of a Polynomial Equation for the Natural Earth Projection

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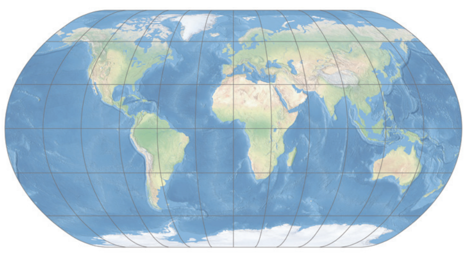
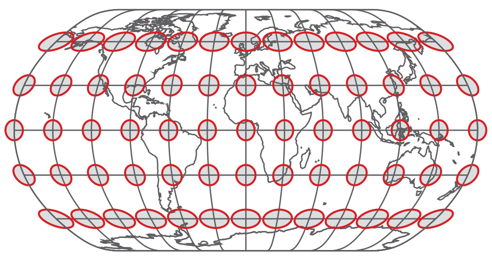
The Natural Earth Projection

The Natural Earth projection was developed in Flex Projector by Tom Patterson (U.S. National Park Service). Using a graphical design approach, he defined the lengths and the vertical distribution of parallels for every five degrees of increasing latitude. In the Flex Projector implementation of the projection, cubic spline interpolation determines the position of intermediate points.

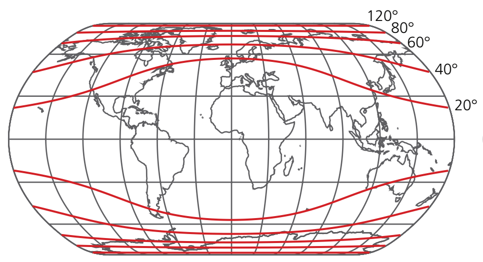
This true pseudo-cylindrical projection has a distinguishing characteristic – rounded corners where border meridians meet the pole lines. The Natural Earth projection is neither conformal nor equal area, but has distortion characteristics comparable to other well-known projections.



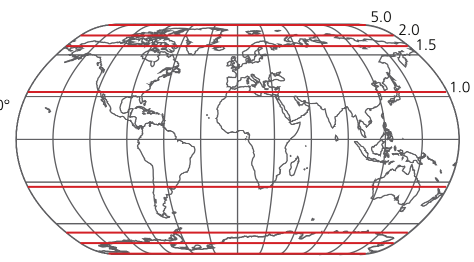
Natural Earth



Tissot's indicatrices



Isocols of angular distortion



Isocols of areal distortion

Goals and Methods

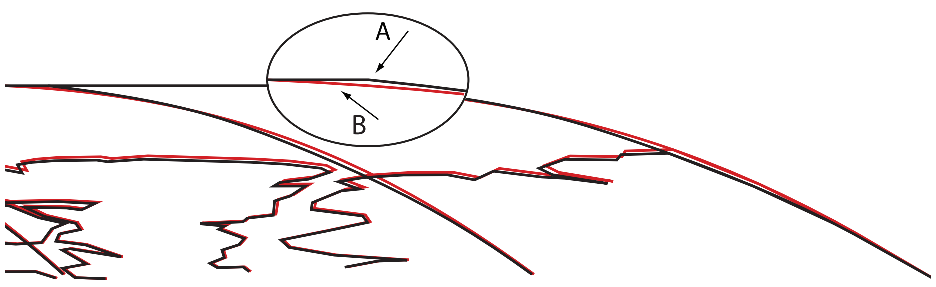
1. Analytical expression for projecting spherical φ / λ to Cartesian X / Y coordinates

A simple polynomial approximation was developed, which involved least square adjustments. Preserving the dimensions of the graticule required the addition of two constraints to these adjustments.

2. Smoothed corners at the end of the pole lines

Shortening the length of the pole lines and reducing the slope of the meridians improved the roundness of the corners.

A (black) – original Natural Eart projection,
B (red) – improved polynomial projection



Polynomial Equations

$$X = R \cdot \lambda \cdot (A_1 + A_2\varphi^2 + A_3\varphi^4 + A_4\varphi^{10} + A_5\varphi^{12})$$
$$Y = R \cdot (B_1\varphi + B_2\varphi^3 + B_3\varphi^7 + B_4\varphi^9 + B_5\varphi^{11})$$

where:
- X and Y are the projected coordinates,
- φ and λ are the latitude and longitude in radians,
- R is the radius of the generating globe,
- A_1 to A_5 and B_1 to B_5 are coefficients given below:

Coefficients for X		Coefficients for Y	
A_1	0.870700	B_1	1.007226
A_2	-0.131979	B_2	0.015085
A_3	-0.013791	B_3	-0.044475
A_4	0.003971	B_4	0.028874
A_5	-0.001529	B_5	-0.005916

For inverting the projection, Newton's method is used for computing the latitude φ from the Y coordinate. The longitude λ is then calculated from the X coordinate.

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